

## Exercise 1

A population of protozoa develops with a constant relative growth rate of 0.7944 per member per day. On day zero the population consists of two members. Find the population size after six days.

### Solution

Use the fact that the relative growth rate is 0.7944 per member per day.

$$\frac{1}{P} \frac{dP}{dt} = 0.7944$$

Rewrite the left side using the chain rule.

$$\frac{d}{dt} \ln P = 0.7944$$

The function that you take a derivative of to get 0.7944 is  $0.7944t + C$ , where  $C$  is any constant.

$$\ln P = 0.7944t + C$$

Exponentiate both sides to get  $P$ .

$$\begin{aligned} P(t) &= e^{0.7944t+C} \\ &= e^C e^{0.7944t} \end{aligned}$$

Use a new constant  $A$  for  $e^C$ .

$$P(t) = Ae^{0.7944t} \tag{1}$$

On day zero the population is 2.

$$P(0) = Ae^{0.7944(0)} = 2 \quad \rightarrow \quad A = 2$$

Consequently, equation (1) becomes

$$P(t) = 2e^{0.7944t}.$$

Therefore, after 6 days the protozoa population is

$$P(6) = 2e^{0.7944(6)} \approx 235.$$